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FM Spacing Requirement for Amateur Transceivers

This is a theoretical look at the spectral width of typical Amateur Radio FM transmissions and what that implies for repeater channel spacing. This short monograph is a personal opinion and analysis and does not represent the opinion or position of the American Radio Relay League, nor is it written in my capacity of Technical Coordinator for the ARRL East Bay Section.

Introduction

There have been many suggestions to narrow repeater channel spacing from the current 20 kHz to 15 kHz or even 12.5 kHz. These narrow channel spacings may present certain problems with adjacent channel interference. Potential interference issues are directly predictable from signal processing theory and what it has to say about energy that will be present in the modulated signal at some distance from the carrier frequency. This discussion is further clouded by the presence of several different modulation schemes, however, some prediction is possible.

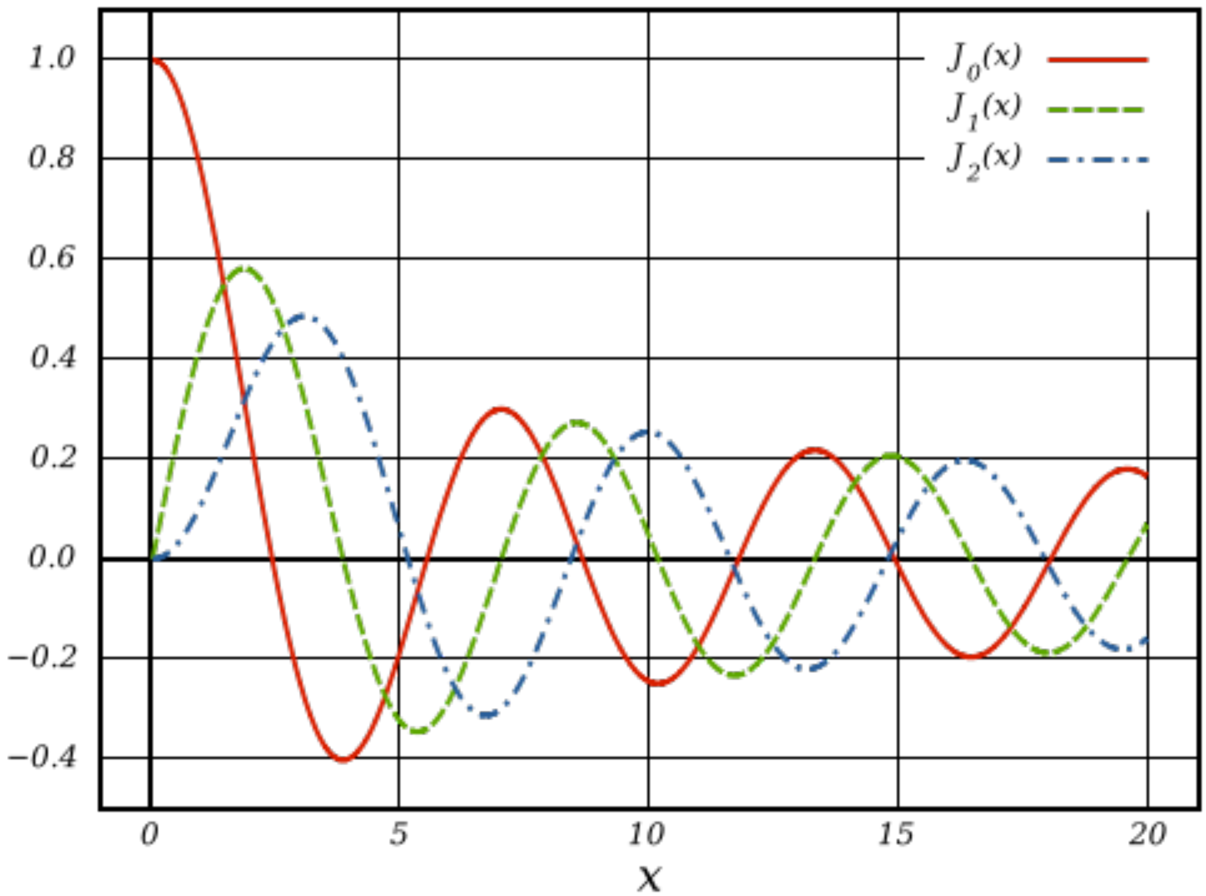
Frequency Modulation

Frequency modulation, or FM, is produced by varying a carrier frequency with desired amplitude changes in the baseband signal to be modulated. Skipping over the mathematical derivation of frequency modulation, the end result, when viewed in the frequency domain, is a type of Bessel function. These Bessel functions have the property that, while they do approach zero at infinity, they contain significant amplitude near the carrier frequency that decreases with distance from the carrier. For this reason, and unlike amplitude modulation, it is not possible to define a specific bandwidth into which all of the amplitude of an FM signal will be contained. Instead, we must speak of the amount of the signal amplitude or energy contained within a specified part of the spectrum.

The amplitude contained within a given portion of the spectrum near the carrier frequency is a function of the modulation index of the signal. The modulation index is defined as the peak frequency deviation divided by the highest frequency component of the baseband.

$$\text{Modulation Index} = \Delta f / f_m$$

The modulation index can be related to the relative amplitude in each of the N orders of the Bessel function that represents the spectrum of the modulated waveform.



FM spectral content is like a Bessel function of the first kind (image from Wikipedia)

Qualitative vs. Quantitative Measure

This mathematical property of FM makes arguments about occupied bandwidth more qualitative than quantitative. In order to speak of bandwidth and be precise, it is necessary to specify both bandwidth and amplitude/power. When those two quantities are not specified, it is not possible to understand the implications of using a given channel spacing. Further complicating this issue, baseband signals will necessarily have non-uniform spectral content and this will affect the precise form in which the modulated spectral energy will be distributed.

Baseband Signals

For Amateur Radio repeater use, baseband signals will typically take two forms: amplitude encoded audio (analog audio) and a relatively new modulation scheme defined by the JARL's D-STAR system. These two baseband signals have unique characteristics. The D-STAR waveforms are appropriately discussed in the context of FM as the D-STAR modulation scheme is to generate a waveform using Gaussian

Mean Shift Keying, or GMSK, and then to use that as the baseband for an FM modulator. This is a quite unusual scheme that results in a wider distribution of spectral energy than might be possible if the GMSK signal was used to amplitude modulate the RF carrier. The GMSK waveform has well distributed spectral content and thus it can be expected to maximize the spectral energy in the FM Bessel sidebands.

In addition to the two baseband schemes, the audio baseband also can use two different modulation indexes. Typical amateur transmissions use a deviation of 5 kHz with a maximum bandwidth of about 5 kHz for a modulation index of approximately one. This is typically called narrow-band FM. There are some transceivers that can be set for 2.5 kHz deviation, lowering the modulation index to ~0.5. Confusingly, this is often called the 'narrow' mode on transceivers. This paper will refer to the modulation indexes to minimize confusion. The D-STAR modulator uses 2.5 kHz modulation or a ~0.5 modulation index to remodulate the GMSK waveform.

It is worth mentioning that as the modulation index of the waveform decreases, the noise immunity decreases as well. Thus a 1.0 modulation index waveform will be more robust against adjacent channel interference when compared to a 0.5 modulation index. It is also worth noting that only a subset of Amateur Radio transceivers are capable of 0.5 modulation index waveforms.

Half-Power Bandwidth

When speaking of bandwidth of FM signals it is not uncommon for specifications to be written by specifying the half-power bandwidth (sometimes called 6 dB bandwidth) of the signal. This is often confused with the notion of a contained bandwidth for the entire modulated signal. The two ideas are not equivalent. For example, D-STAR is often quoted as occupying only 6.25 kHz based upon the specified half-power bandwidth of 6.25 kHz. That specification only implies that half of the spectral energy is contained within 6.25 kHz. The balance of the spectral energy is outside this bound around the carrier frequency and has the potential to cause interference from energy in the additional Bessel sidebands.

Carson's Rule

Given that the spectral energy is spread out and unbounded, it was necessary for engineers to create 'rules of thumb' to define some approximation of the concept of bounded bandwidth. Carson's bandwidth rule is one such approximation that was introduced by John Carson in a 1922 paper at the IRE. Carson's rule states that the *approximate* bandwidth occupied by an FM modulated waveform is 2 times the sum of the peak deviation and the highest baseband frequency.

$$\text{Carson's bandwidth} = 2(\Delta f + f_m)$$

In the case of typical Amateur baseband modulation with a modulation index of 1, the Carson's bandwidth is 20 kHz. For modulation with a 0.5 modulation index, the Carson's bandwidth is 15 kHz.

FM Spectral Distribution

Typical FM spectral amplitude is distributed according to the Bessel function mentioned earlier. Using the Bessel function, a model can be created of the sideband distribution of amplitude. Each sideband is defined as centered an increment of the peak frequency of the baseband away from the carrier frequency. Using the Bessel model, the energy is distributed as follows for modulation indexes of 0 (no modulation), 0.5, and 1. The total amplitude is 1.0.

carrier	1st sideband	2nd sideband	3rd sideband	4th sideband
Modulation index 0	1.0	0	0	0
Modulation index 0.5	0.94	0.24	0.03	0
Modulation index 1.0	0.77	0.44	0.11	0.02

From this table it can be seen that for a 5 kHz baseband signal with a modulation index of 1, approximately 11% of the carrier amplitude (-19.2 dB) is 15 kHz away from the carrier frequency. If the modulation index is 0.5, only 3% of the amplitude (-30 dB) is 15 kHz away from the carrier frequency. This correlates well with the results from Carson's rule for both the 0.5 and 1 modulation indexes, if we assume that greater than approximately -25 dB amplitude from an adjacent channel represents significant interference.

Implications for Channel Spacing

It is not possible to state precisely that a particular channel spacing will be free of significant interference when speaking of FM waveforms. It is, in the author's opinion, important to realize that interference can be calculated for a specific situation and each application's performance with an average set of test cases will be affected by choices of channel spacing.

Strictly using Carson's rule, a 20 kHz channel spacing would be recommended for current Amateur applications where a modulation index of 1 is in use. If it were possible to move all communications to a modulation index of 0.5 (this is likely not practical given the installed base of transceivers) then a 15 kHz channel spacing would be recommended by Carson's rule.

Using the Bessel model of FM and the spectral distribution of signal amplitude generated from the model, similar recommendations obtain. Suppression of signal amplitude to -30 dB is predicted by the model at the channel spacing recommended by Carson's rule.

Conclusions

The signal processing implications of modulation waveforms are well understood. Signal processing theory accurately predicts spectral components of carrier signals that are modulated by various schemes. For frequency modulation, the modulation index and the baseband peak frequency predict the signal amplitude in the sidebands of the modulated waveform.

The spectrum of FM waveforms is unbounded and thus channel spacing in this context must be looked at qualitatively with quantitative data as a guide. These theoretical models and the quantitative data they produce can be used to guide decisions about channel spacing.

Both Carson's rule and the Bessel model of FM spectral content recommend channel spacing of 20 kHz for signals with a modulation index of 1, such as typical Amateur Radio voice modulation. They also recommend channel spacing of 15 kHz for signals with a modulation index of 0.5, such as 'narrow' analog voice and D-STAR's GMSK-FM modulation.

These recommendations are not absolutes. They reflect many assumptions about noise tolerance of the channel and waveform demodulator. In some cases, they may represent minimums. D-STAR, in particular, with it's wide spectral distribution in the baseband, may be problematic as it will maximize the amplitude of the Bessel sidebands. Appropriate caution and testing is advised.